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THE MOBILITIES OF THE POSITIVE IONS IN HELIUM

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Early investigations in the field of ionic conduction of electricity in gases revealed that the mobility computed from simple kinetic theory considerations was much higher than the values actually observed. This led to the hypothesis that the ions consist not of single charged molecules, but of clusters of molecules. Langevin[1] computed the size of the cluster that exists in thermal equilibrium with the surrounding gas, and found it to consist of from six to ten molecules, assuming the ion to be a point charge and the molecules surrounding it to be polarized conducting spheres.

Since this earlier work, investigators have at various times attempted to prove or disprove the cluster theory, and as a consequence a great deal of evidence has been presented for both sides. It is only quite recently through the work of Erikson[2] and Nolan[3] that the evidence points unquestionably to a cluster theory. For instance, Erikson using a modification of the Zeleney air-flow method for determining mobilities, has found that the value of the mobility observed depends on the time that elapses beween the formation of the ion and the instant it enters the field in which the mobility is measured.

Thus, if the mobility of the positive ion in air was determined immediately after the formation of the ion, the value found was 1.89 cm./sec./volt/cm., and if it was aged for about one half of a second after the measurements were made, the value found was that usually ascribed to the positive ion, viz., 1.35 cm./sec./volt/cm. The writer[4] was able to check Erikson's results using the alternating current method developed by Rutherfords, and modified by Franck.

The work presented in this paper was undertaken to see if this aging in the mobility could be obtained in a monatomic gas. The method of procedure was the same as that used in the paper referred to above. *G* (Fig. 1) was a gauze to the lower side of which a brass cup was fastened, and in this the source of ionizing radiation (in this case a copper plate

coated with polonium), was placed. The cup was of sufficient depth so that the upper part of the region between the gauze and the metal plate P was shielded from the radiation. A source of potential was connected between G and P. This could be varied from $^1/_2$ a volt to 27 volts. Thus the ions could be driven through the gauze immediately after they were formed, or could be allowed to age before entering the measuring field. The alternating potential was applied to the gauze, and was obtained from an audio frequency vacuum tube oscillator of the Hartley type. The plate C served to collect the ions driven across the space between C and G by the alternating field. The charge received by C could then be measured by the electrometer E. The gauze was 10 cm. in diameter, and was made by drilling holes in a sheet of brass 0.35 mm. thick. The holes were 1 mm. in diameter, and the distance between adjacent holes 4 mm. The distance between P and G was 1 cm. and the distance between B and C was varied from 0.9 cm. to 1.5 cm. The cup A had an outside edge of approximately 5 mm. so that the ions had to pass through at least this thickness of unionized gas before entering the measuring field.

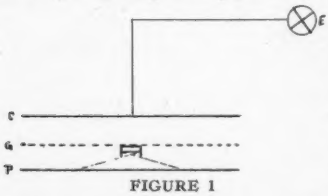

FIGURE 1

The method of procedure was the usual one of measuring the current to C for different values of the alternating potential, and from the curve obtained by plotting the current against voltage, determine by extrapolation the potential at which the ions ceased getting across to C.

The mobility K was then determined from the relation:

$$K = \frac{\pi n d^2}{\sqrt{2}V} \frac{P}{760}$$

where P is pressure, n is the frequency of the alternating potential, d is the distance between the gauze and the collecting plate, and V is the effective value of the voltage corresponding to the intercept of the mobility curves corrected for the change in potential of the collecting plate when a potential is applied to the gauze.

The helium was purified by passing it through two traps containing charcoal and immersed in liquid air. The charcoal had been partially activated with steam, and was baked out at a temperature of 300°C for a period of several hours before being immersed in liquid air. At intervals the helium was withdrawn from the chamber in which the gauze and plates were placed and repurified. In this way any impurities which might diffuse into the gas from the walls of the vessel were removed.

Mobilities were determined at pressures ranging from 10 to 35 cm., and with auxiliary fields of from $^1/_2$ to 27 volts/cm.

Figure 2 illustrates the types of curves obtained with auxiliary fields if 18 volts/cm. (curve 1) and 1.5 volts/cm. (curve 2) at a pressure of 10 cm., and with a frequency of 662 cycles/sec. The plate distance d in this case was 9.5 mm. Thus we see that changing the auxiliary field from 1.5 to 18 volts per cm. all other conditions remaining the same increases the mobility from the value corresponding to the intercept at 54 volts to the value corresponding to the intercept at 28 volts. These curves may be taken as representative of those obtained for the completely aged (2) and for the unaged (1) ions, since a lowering of the voltage below 1.5 or an increase above 18 produced no further material change.

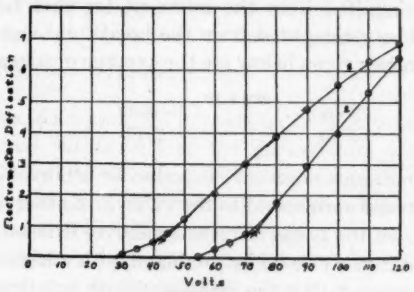

FIGURE 2

These mobility curves differ from those ordinarily obtained in that they show breaks at 73 and 43 volts. These same breaks were present in all the curves obtained in helium. They might be due to either of two causes: In the first place if any irregularity existed in the gauze so that the distance between it and the collecting plate was not uniform, the mobility curves would show a tendency to approach the voltage axis less sharply. However, with increased plate distance, this tendency should become less. This was not found to be the case. Also the curves obtained for the unaged ions in air should show the same tendency to concave upward, if it were due to an irregularity in the distance between the gauze and plate. Most of the curves obtained in air showed no sign of a break, and when it was present it was so slight as to be negligible.

The other possible explanation of the presence of breaks is that they are due to the presence in helium, under the influence of α particle ionization of two types of aged and two types of unaged ions having different mobilities. That they are not due to a mixture of the aged and unaged ions is indicated by the fact that increasing the time of aging for the aged ions or decreasing the time of aging for the unaged ions did not cause a disappearance of the breaks.

There is a possibility that the two types of ions are the singly charged and the doubly charged ions observed by Millikan[5] by the oil drop method. In a recent paper, however, Loeb[6] gives an expression for the mobility of an ion which does not involve the charge on the ion. According to this equation, it should then be impossible to distinguish between a singly charged and a multiple charged ion.

Unfortunately the ions corresponding to the part of the mobility curves

below the break were present in small numbers so that it was impossible to determine the intercept with any degree of accuracy. The mobilities computed from the intercept for that reason varied over a 25% range. Those computed from the breaks did not show so great a variation. The values given below are the average of seventeen determinations:

Aged Ions		Unaged Ions	
Break	Intercept	Break	Intercept
5.13	7.17	8.70	13.30

Franck observed one value for helium, viz., 5.09 cm./sec./volt/cm. This would correspond to the value 5.13 observed by the writer.

In the paper referred to above, Erikson showed that the mobility of the ions in air could be predicted with a high degree of accuracy provided it is assumed that the mean free path is independent of the nature of the ion, and depends only on the gas through which the ion moves, and if it is assumed that the value 1.82 cm./sec./volt/cm. is that of a single charged molecule. With this assumption he shows that the mobility should vary inversely as the square root of the mass of the ion. Loeb in the paper mentioned above also concludes that the free path should be independent of the nature of the ion, and the mobility in any given gas should vary only with the mass.

If the assumptions that Loeb makes in his derivation are justified, the maximum variation in the mobility that one could expect would be such that the ratio of the mobility of a single-molecular ion to that of even a very heavy ion should not exceed $\sqrt{2}$. Therefore, the ratio of the mobility of an unaged ion to the mobility of the corresponding aged ion should not exceed this value. If we take the ratio as determined from the breaks in the mobility curves, we see that it has a value 1.696 which is more nearly equal to $\sqrt{3}$ which, according to the assumptions made by Erikson, means that the aged ion would consist of three atoms. If this is the case, it should then be possible to obtain a two atom cluster, i.e., an intermediate stage between the unaged and completely aged ions. Due to the presence of the breaks in the mobility curves, this type of ion, if present, was so obscured that the writer was unable to obtain any evidence for its existence or non-existence.

[1] Langevin, P., *Ann. d. Chim. e. d. Phys.* (VII) **28,** p. 245 (1903).
[2] Erikson, H. A., *Physic. Rev.*, **20,** p. 117 (1922).
[3] Nolan, J. J., *Proc. Roy. Irish Acad.*, **36, A,** 2, p. 31; **A,** 5, p. 74.
[4] Wahlin, H. B., *Physic. Rev.*, **20,** 3, p. 267 (1922).
[5] Millikan, R. A., *Physic. Rev.*, N. S., **18,** p. 456 (1921).
[6] Loeb, L. B., *Phil. Mag., London*, Sept. 1924, p. 446.

THE DENSITY OF OXYGEN

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The study of gas densities is one of interest and importance because from these constants many fundamental values such as the gas constant and molecular (and atomic) weights may be calculated. The most important gas density is that of oxygen since it is commonly employed as the standard gas in the application of Avogadro's Rule. Although the density of oxygen has been experimentally determined by some of the ablest scientists, including Regnault, Lord Rayleigh and E. W. Morley, as well as several experimenters in the laboratory of the late P. A. Guye at Geneva, uncertainty has been introduced by recent experiments and papers by Moles[1] and collaborators who find a value distinctly lower than the one which has been in general use for some time.

Our work offers little that is novel in this field of experimentation. The study of earlier investigations has been of the greatest assistance in suggesting methods of operation and indicating the difficulties likely to be met in making the various observations. On the whole, however, we believe that we have succeeded in increasing the refinement of measurements over that of earlier work. Furthermore our results show a much greater consistency among themselves than in any previous similar investigation, without, however, differing markedly from the value obtained by the most reliable investigators.

The determination of gas density requires the measurements of weight, volume, temperature and pressure.

The globes, three in number, of about 1000 ml. interior volume were weighed by direct comparison with a sealed, exhausted counterpoise of similar exterior volume, on a balance of large capacity and high sensitiveness. Since the globes both when exhausted and when filled with oxygen were lighter than the counterpoises, the weight of oxygen was found by differences of weights placed on the same side of the balance as the globes, so that the oxygen was weighed by substitution. Temperature disturbances were avoided in part by insulating the balance case with a cover of plumbers felt. Observations were made through small double glazed windows. The best results were obtained only after thermostating the balance room to 1°. The balance case was provided with a small amount of impure radium bromide to dissipate electrostatic charges. Under these conditions the weights of the globes in a short time became constant within a very few hundredths of a milligram and remained so indefinitely.

The interior volumes of the globes were found in the usual way by

weighing them, at room temperature, empty and after filling them with water at 0°. The volumes were found not to alter during a period of six months. Temperature control while the bulbs were filled with water was within 0.02°. Weighings were made by substitution.

The globes were filled with oxygen when immersed in a bath of washed ice and distilled water which was thoroughly stirred. In each experiment the temperature of this bath was observed with a sensitive thermometer and was found to be very constant, nearly always slightly below 0°, but never lower than —0.026°. Allowance was made for this difference in the calculation of density.

The most difficult measurement to make was that of pressure. This was done by means of mercury barometers communicating directly with the globes. Readings were made by setting the cross hairs of two horizontal telescopes of a Geneva cathetometer on the mercury levels, and then by rotating the cathetometer through a small angle observing the readings of the cross hairs on a standard meter bar. This bar had been calibrated by the United States Bureau of Standards some years ago, and a calibration after our experiments were completed shows it not to have altered in length materially in the interval.

The temperatures of the barometers and the meter bar were controlled as carefully as possible. In the first place the room in which the experiments were carried out was maintained constant in temperature within a degree by a thermostatic device and fan. The meter bar was surrounded with a wooden box the interior temperature of which was observed with two thermometers. The barometer used in the first six experiments (No. 1) was 20 mm. in diameter and was located in a wooden box with openings for observations. In addition a second barometer (No. 2) was attached for the next six experiments. This was 40 mm. in diameter and was enclosed in a water jacket which was thoroughly stirred. In the five following experiments the air-jacketed barometer was discarded and a third water-jacketed barometer (No. 3) like the second was attached. Still closer control of the temperature was secured in the final five experiments by circulating water at constant temperature through the water jackets of the barometers (Nos. 4 and 5). Except in the case of No. 1, the various barometers gave consistent readings within 0.01 mm., so that this quantity probably represents the accuracy of the pressure measurement. Barometer readings were corrected to 0° and for the difference in the force of gravity at the Coolidge Laboratory and at sea level in latitude 45°.

Usually the pressure at which the globes were filled differed only slightly from 760 mm. In a few of the earlier experiments where the difference was greater, allowance was made for this fact by means of the coefficient of deviation from Boyle's Law.

Contraction of the globes when exhausted was found both by the Crafts method and by the Lord Rayleigh method. Essentially identical results were obtained, corresponding closely to the Lord Rayleigh-Moles formula:

$$\text{Contraction} = 15.5 \frac{\text{volume in milliliters}}{\text{weight in grams}} 10^{-3}$$

Oxygen was prepared (1) by electrolysis of dilute sulphuric acid, (2) by electrolysis of aqueous sodium hydroxide, (3) by heating potassium permanganate, (4) by heating a mixture of potassium chlorate and manganese dioxide. All the oxygen was purified by passing over aqueous potassium hydroxide, solid fused potassium hydroxide, red-hot platinum gauze, amalgamated copper and phosphorus pentoxide. In addition the permanganate and chlorate oxygen was passed through dust removers and the chlorate oxygen over hot silver wire. Furthermore in the last thirteen experiments the oxygen was liquefied and fractionally distilled three times before the bulbs were filled.

The generating, purifying and filling apparatus was constructed throughout of glass and quartz with fused or ground connections. In the early experiments the globes were attached by rubber connections, but because of suspected leakage ground connections were used after the fourth experiment.

Mercury was purified by prolonged treatment with an acid solution of mercurous nitrate and was three times distilled in an air current at low pressure. The portions rejected were small so that there was no danger of material change in isotopic composition in the distillation process.

The outline of the experimental procedure is as follows: The three globes were exhausted to a pressure below 0.005 mm. and after being wiped first with a moist then with a dry cloth were weighed. They were then attached to the oxygen apparatus and while immersed in the ice bath were slowly filled with oxygen. After the whole system had been allowed to stand for several hours with all the thermostats operating, the barometers were read and the globes were immediately closed. As soon as the globes had been weighed, filled with oxygen, they were again exhausted and weighed. The average weight of the exhausted globe before and after filling with oxygen was used in calculating the density.

The following constants are employed:

Density of water at 0°	0.999868
Cubical coefficient of expansion of mercury	0.0001818
Coefficient of expansion of oxygen	0.003672
Coefficient of deviation from Boyle's Law per atmosphere	—0.00097
Force of gravity at latitude 45° and sea level	980.616
Force of gravity at Coolidge Laboratory	980.376

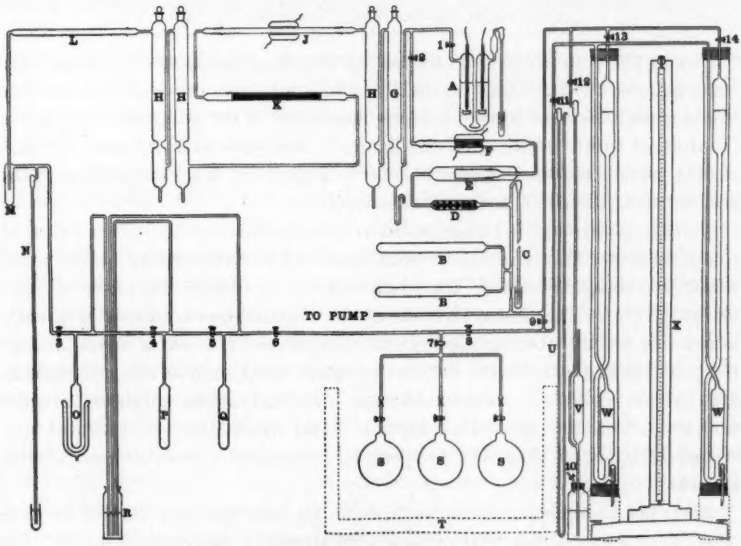
THE DENSITY OF OXYGEN

0° AND 760 MM. AT SEA LEVEL LAT. 45°

NUMBER OF EXPERIMENT	SOURCE OF OXYGEN	BAROMETER NO.	GLOBE I	GLOBE II	GLOBE III	AVERAGE
		Series 1				
1	Electrolytic	1	(1.42726)	1.42899	1.42869	1.42884
2	Electrolytic	1	1.42949	1.42803	1.42820	1.42857
3	Electrolytic	1	1.42883	1.42939	1.42910	1.42911
4	Electrolytic	1	1.42847	1.42883	1.42866	1.42865
		Average	1.42893	1.42881	1.42866	1.42879
		Series 2				
5	Electrolytic	1	1.42939	1.42938	1.42933	1.42937
6	Electrolytic	1	1.42926	1.42904	1.42918	1.42916
		Average	1.42933	1.42921	1.42926	1.42926
		Series 3				
7	Electrolytic	1 and 2	1.42887	1.42892	1.42889	1.42889
8	Electrolytic	1 and 2	1.42926	1.42906	1.42926	1.42919
9	Electrolytic	1 and 2	1.42942	1.42924	1.42944	1.42937
		Average	1.42918	1.42907	1.42920	1.42915
	Average of Series 1, 2, 3		1.42912	1.42899	1.42897	1.42902
		Series 4				
10	Electrolytic, distilled	1 and 2	1.42899	1.42887	1.42870	1.42885
11	Electrolytic, distilled	1 and 2	1.42917	1.42915	1.42929	1.42920
12	Electrolytic, distilled	1 and 2	1.42895	1.42898	1.42909	1.42901
		Average	1.42904	1.42900	1.42903	1.42902
		Series 5				
13	Electrolytic, distilled	2 and 3	1.42897	1.42893	1.42924	1.42905
14	$KMnO_4$, distilled	2 and 3	1.42890	1.42907	1.4290[illegible]	[illegible]901
15	$KMnO_4$, distilled	2 and 3	1.42906	1.42902	1.42918	[illegible]09
16	$KMnO_4$, distilled	2 and 3	1.42881	1.42884	1.42897	1.42887
17	$KMnO_4$, distilled	2 and 3	1.42892	1.42895	1.42903	1.42897
		Average	1.42893	1.42896	1.42911	1.42900
		Series 6				
18	$KClO_3$, distilled	4 and 5	1.42895	1.42901	1.42893	1.42896
19	$KClO_3$, distilled	4 and 5	1.42900	1.42900	1.42905	1.42902
20	$KClO_3$, distilled	4 and 5	1.42899	1.42894	1.42905	1.42899
21	$KMnO_4$, distilled	4 and 5	1.42897	1.42903	1.42900	1.42900
22	Electrolytic, distilled	4 and 5	1.42906	1.42903	1.42906	1.42905
		Average	1.42899	1.42900	1.42902	1.42900
	Average of all experiments		1.42903	1.42899	1.42902	1.42901
	Average of Series 4, 5, 6		1.42898	1.42899	1.42905	1.42901

No experiments are omitted from the foregoing table. Series 1 was subject to uncertainty owing to rubber connections used to attach the globe. In Series 1 and 2 only the first rather unsatisfactory barometer was employed. In Series 1, 2 and 3 the oxygen was not liquefied and fractionated. For these reasons the results of Series 4, 5 and 6 are more reliable than the earlier ones. The average of the experiments in these series is, however, identical with the average of all.

The density found, 1.42901, is very slightly lower than the one in general use, 1.42905. The latter value has been discussed critically by Moles and found by him to be subject to a correction of —0.00014.[2] The experimental value found recently by Moles and his collaborators is 1.42892.[1b]



A	One of six electrolytic cells
B, B	Decomposition tubes for permanganate and chlorate
D	Dust trap filled with glass pearls
E	Porous earthernware dust filter
F	Hard glass tube containing silver wire
G	Potassium hydroxide solution
H, H, H	Solid potassium hydroxide
J	Platinum gauze
K	Amalgamated copper
L	Resublimed phosphorus pentoxide
O, P, Q	Liquefaction tubes
T	Ice bath for density globes
S, S, S	Density globes
V	McLeod gauge
W, W	Barometers
X	Meter bar

Using the density 1.42901 and the coefficient of deviation from Boyle's Law per atmosphere, —0.00097, the limiting value of the molal volume of oxygen calculated to standard conditions becomes 22.415 liters.

[1] Moles and Batuecas, *J. chim. phys.*, **18**, 353 (1920); Moles and Gonzalez, *ibid.*, **19**, 310 (1921); Moles and Crespi, *Anales soc. espan. fis. quim.*, **20**, 190 (1922).

[2] Moles, *J. chim. phys.*, **19**, 100 (1921).

OBSERVATIONS ON THE FAUNA OF THERMAL WATERS

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During the summer of 1923 the writer had the opportunity to make some observations on the fauna of the thermal springs in the Yellowstone National Park which led him to become interested in the adaptations of aquatic animals to life under such conditions. The present brief note summarizes a more extensive account soon to appear in the Proceedings of the American Academy of Arts and Sciences.

Certain plants and animals occur regularly in thermal waters living at temperatures either slightly or sometimes very considerably above those which are compatible with the continued life or normal existence of most forms of life. The characteristics of the thermal environment of aquatic organisms are the abnormally high temperature of the water in which they live; its impregnation with various inorganic salts such as calcium carbonate, sodium carbonate, sodium chloride, silica and certain sulphates in quite noticeable amounts and other salts in lesser quantities; and almost universal deficiency in dissolved oxygen and frequently a great excess of carbon dioxide.

All types of aquatic organisms present are affected very directly by the heat of the water as the heat of their own tissues is determined by it. The plants are affected greatly by the varying amounts of oxygen and carbon dioxide, and those animals that are dependent upon dissolved oxygen for respiration must adapt themselves to the dearth of this essential element.

In Yellowstone Park and in other thermal springs plant life exists in water of extremely high temperatures, varying considerably in this respect, dependent in great measure upon the acidity or alkalinity of the water. Species without chlorophyll exist in water of 70°–89° C. and green algæ at 60°–77° C.

Animal life extends into water of from 46°–52° C., although but few species are found above 40°–42° C.

The composition of the present thermal fauna which is extremely varied, includes Protozoa, Arthropoda, Mollusca and cold blooded Vertebrates in sufficient numbers to indicate its probable origin. One might assume that it is derived from a primordial thermal fauna since the beginnings of life were undoubtedly in warm or hot water, but the diversity of the present thermal types which include highly specialized insects and molluscs closely related to dominant cool water forms shows this supposition to be erroneous although it may be true that certain types of very primitive plants are derived directly from a primordial thermal flora.

Experiments by numerous biologists have shown that marine animals accustomed to uniformly cool water are most susceptible to the heating of the medium and that they ordinarily succumb to temperatures of from 25°-30°C. Littoral forms as might be expected from the conditions of their normal habitat are somewhat more resistant and fresh-water animals still more so. The latter, of course, frequently find it necessary to withstand considerable rises of temperature extending over more or less protracted periods. On this basis we might believe that the thermal fauna has been derived piecemeal from fresh-water and this is borne out by the types of animals of which it is composed.

There is, however, a striking similarity between the thermal and brackish water faunæ, but it is apparent on examination that this is due to the presence of the same types in each, derived generally in both cases from fresh-water. Thus we find among the insects that the genera of aquatic beetles characteristically represented in thermal waters have also migrated into and become established in brackish water and the same is true of several families of Diptera, certain crustaceans and molluscs. It would appear that the similarity of these two faunæ is due to the powers of resistance to salts in solution of which there is an excess in thermal waters also.

It is quite evident that the animals characteristic of thermal waters have become acclimatized to withstand temperatures that are highly detrimental or even fatal to normal animals but the range over which acclimatization is possible is by no means great and the upper limit is moreover very nearly uniform for all animals. This upper limit is in the neighborhood of 40°C. (104° Fahr.) or a few degrees higher, and corresponds closely with the thermal death point of mammals, including man which like the aquatic thermal animals live precariously near this point. The invertebrate fauna of the desert appears also to have practically the same temperature requirements.

The physiological adjustment by which acclimatization becomes possible is not clear, but experiments with specific animals and the uniformity of temperature requirements among diverse animals make it seem very probable that high temperatures act directly upon the protoplasm or cell metabolism of animals.

Specifically the aquatic thermal fauna includes a number of protozoans and minute invertebrates which have been but little studied, a quite considerable series of insects and molluscs and a few cold-blooded vertebrates.

Noticeably among the insects are a number of species of aquatic beetles and their larvæ, some of which occur in both thermal and meteoric waters, some extend their range considerably into cooler regions by availing themselves of thermal waters and still others appear to be peculiar to warm springs.

Several small chironomid flies are also typical inhabitants of thermal waters and among these the blood-worms of the genus Chrionomous occur in water practically devoid of oxygen, the traces of which they are able to utilize through the action of the hæmoglobin present in their blood.

The larvæ of certain Stratiomyid flies and tabanids occur in Yellowstone Park, the former elsewhere also in thermal waters, not affected by the scarcity of oxygen since they obtain their air from the atmosphere above the surface of the water. Larvæ of another fly, Ephydra, characteristic of saline and brackish waters, inhabit thermal water as well.

Several other orders of insects, including may-flies, caddice flies, dragon-flies and Hemiptera, are less abundantly represented, including some casual and some truly thermophilous species.

One abundant crustacean, *Gammarus limnæus* known from the arctic and high mountain lakes extends into the thermal waters of the Park.

Among the molluscs, several genera, e.g., Physa and Limnæa extend into thermal waters.

Even frog tadpoles were found in Yellowstone Park in water somewhat above the lethal temperature for unacclimatized individuals.

ON THE OCCURRENCE IN RABBITS OF LINKAGE IN INHERITANCE BETWEEN ALBINISM AND BROWN PIGMENTATION

By W. E. Castle

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Read before the Academy, November 12, 1924

The dark pigment of a rabbit's coat may be either black or brown in color. The two conditions are mutually exclusive. The coat is prevailingly either black or brown, and as regards inheritance, black is dominant over brown.

A brown race which I obtained a few years since for experimental study produced about twenty-five per cent of albino young, of the sort known as Himalayan. These were pink-eyed and white-coated except at the extremities where the fur was brown pigmented. It was evident that albinism of the Himalayan type was present as a recessive character in my race of brown rabbits. Hence its appearance in one-fourth of the young, when brown was mated with brown. The Himalayan young produced in this way would necessarily be pure (homozygous) for the two recessive characters, (1) *brown* and (2) *Himalayan albinism*. It is my purpose to show that these two characters are linked in inheritance.

When the brown Himalayan rabbits were mated with black rabbits,

black offspring were obtained, both of the recessive characters having disappeared as expected in the presence of their dominant allelomorphs, black and full color. In the second or F_2 generation, about 25 per cent of Himalayan young were produced as expected. Some of these were brown Himalayans and others were black Himalayans, and the natural inference was made that the two pairs of characters were independent of each other in inheritance and so assorted freely.

Thus the matter stood until a renewed study of the case was made in a systematic search for linkage among the inherited characters of rabbits. For this purpose back-crosses of F_1 with double recessive individuals are most useful, since in such matings four classes of young are expected all equally numerous, and it is easy to detect consistent deviations from equality in these classes, such as linkage would produce.

The double recessive in this experiment would be a brown Himalayan individual, the F_1 would be a black individual heterozygous both for brown and for albinism. Such individuals in part were used in the back-crosses, but part of the F_1 individuals used were derived from a cross of chinchilla with brown Himalayan, chinchilla being another allelomorph of Himalayan albinism, dominant to it but recessive to full color.

In the back-crosses, whatever albino allelomorph is involved in the F_1 parent, as a rule *brown Himalayans* are in excess of *black Himalayans*, and *black young of full color* (or chinchillas) are in excess of *brown young of full color* (or chinchillas). In other words the two pairs of character have a tendency to retain their original associations when they segregate in the gametes formed by F_1 individuals. This is true in the gametes formed by F_1 individuals of both sexes, as will be seen in Table 1.

TABLE 1

INEQUALITY IN THE FOUR CLASSES OF YOUNG PRODUCED BY F_1 PARENTS HETEROZYGOUS FOR BROWN AND FOR HIMALAYAN ALBINISM, WHEN MATED WITH DOUBLE RECESSIVES, BROWN HIMALAYANS

	NUMBER OF YOUNG				
HETEROZYGOUS PARENT	COLORED BLACK	COLORED BROWN	HIMALAYAN BLACK	HIMALAYAN BROWN	TOTALS
Male, chinchilla	17	9	12	24	62
Female, chinchilla	56	46	35	59	196
Female, full color (black)	18	11	14	8	51
Totals	91	66	61	91	309

There have been produced in these back-crosses 309 young, of which 152 were Himalayan. Of these a majority (91) were *brown*, while 61 were black. Of the colored classes, a majority (91) were *black* while 66 were brown.

The minority classes, brown colored and black Himalayan, which represent new combinations first obtained in F_2, and which are accordingly cross-over

classes, form 41.1 per cent of the total, 309. This differs from the 50 per cent expected when no linkage exists by 8.9 per cent. But the probable error for a population of 309 individuals is only 1.92 per cent. The observed difference is therefore 4.6 times the probable error, which shows it can scarcely be due to random sampling, since the chances for a deviation equally great are less than two in a thousand, the odds against it being about 650 to 1. We may conclude therefore that in all probability the two characters are linked.

The notable thing about the case is the looseness of the linkage. In cases of linkage previously demonstrated in mammals, linkage is relatively close, less than 20 per cent of crossing over being found. Here it is more than twice that amount. Measured on a scale of 100, the linkage strength in this case is only 17.8 whereas in other known mammalian cases it ranges from 60 to 99. On account of the small size of mammalian chromosomes, loose linkages are not to be expected among mammals, on the chromosome hypothesis. What the cytological conditions are which are involved in this case, we can at present only conjecture.

Analogous variations to those discussed in this paper, producing brown pigmentation and multiple albino allelomorphs, are known both in mice and in guinea-pigs but in neither of these species has linkage been reported between brown and albinism. The matter is perhaps worthy of further investigation, although it is scarcely to be expected that completely homologous chromosomes exist in forms so widely separated systematically as rabbits, guinea-pigs and mice.

A GENERAL TYPE OF SINGULAR POINT

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The present note deals with a general type of singular points of linear differential equations of the second order. We shall study the representation of the solutions in a certain region abutting upon a singular point. Results regarding the distribution of the zeros of the solutions in such regions will follow.

1. Let $K(z)$ and $G(z)$ be analytic functions of z such that

$$\frac{G(z)}{K(z)}, \quad \frac{G'(z)}{G(z)} \text{ and } \frac{K'(z)}{K(z)}$$

are single-valued, and let the singular points of $K(z)$ and $G(z)$ have only

a finite number of limit-points. The solutions of the self-adjoint differential equation

$$\frac{d}{dz}\left[K(z)\frac{dw}{dz}\right] + G(z)\, w = 0 \tag{1}$$

will be single-valued upon a certain Riemann surface $\mathfrak{R}$.

Now apply the transformation of Liouville

$$Z = Z(z;z_0) = \int_{z_0}^{z} \sqrt{\frac{G(z)}{K(z)}}\, dz, \quad W = [G(z)\, K(z)]^{\frac{1}{4}} w \tag{2}$$

which carries equation (1) over into

$$\frac{d^2W}{dZ^2} + H(Z) \quad W = 0 \tag{3}$$

where

$$H(Z) = h(z) = 1 + [G(z)]^{-\frac{3}{4}}[K(z)]^{\frac{1}{4}} \frac{d}{dz}\left\{K(z)\frac{d}{dz}\left([G(z)\ K(z)]^{-\frac{1}{4}}\right)\right\}. \tag{4}$$

z_0 is an arbitrary non-singular point. Let us put $Z = X + i\, Y$ and imagine that the net $X = \text{const.}$, $Y = \text{const.}$ is traced upon $\mathfrak{R}$.

2. Suppose that $z = z_s$ is a singular point of the differential equation, and suppose that there exists a region D abutting upon z_s which has the following properties:

[1] D is an open simply-connected region on $\mathfrak{R}$, the boundary of which is formed by a countable set of arcs belonging to the X, Y-net.

[2] $z = z_s$ is a boundary point of D and there is no other singular point of (1) in D or on its boundary. Further, $G(z) \neq 0$ in D.

[3] There exists a determination of $Z(z;\, z_0)$ which maps D upon a non-overlapping region Δ in the right half of the Z-plane such that Δ contains the half-plane $X \geqq A > 0$.

[4] There exist directions $\arg Z = \theta$ and a finite quantity a, independent of θ where $a = r\, e^{i\phi}$ and $-\frac{\pi}{2} < \phi < +\frac{\pi}{2}$, such that $H(Z) \longrightarrow a^2$ uniformly if $\mathfrak{R}\ [e^{-i\theta} Z] \longrightarrow +\infty$ in Δ. In particular, we shall be able to take $\theta = -\phi$.

[5] There exists a function $M(u)$ of the real variable u, positive, continuous, monotonic decreasing and integrable over the range $(-\omega, +\infty)$ for any finite ω, such that $|F(Z)| < M(u)$ when $X \geqq A$, where $F(Z) = a^2 - H(Z)$ and $u = \mathfrak{R}\ [e^{i\phi} Z]$. If $a = 0$ we require that $u^2\, M(u)$ shall be integrable over $(-\omega, +\infty)$.

3. It follows from the assumptions in § 2 that equation (3) is asymptotic (in an obvious sense) to the equation

$$\frac{d^2W}{dZ^2} + a^2\, W = 0, \tag{5}$$

provided Z remains in Δ. The solutions of (5) are

$$W_0(Z) = c_1 e^{aiZ} + c_2 e^{-aiZ} \tag{6}$$

if we assume $a \neq 0$. The case $a = 0$ will be considered in §4. It is to be expected that the solutions of (3) will be asymptotic to the solutions of (5) in a certain sense.

Replace equation (3) by the associated integral equation

$$W(Z) = W_0(Z) = \frac{1}{a}\int_Z^\infty \sin a(T-Z)\, F(T)\, W(T)\, dT \tag{7}$$

where $W_0(Z)$ is defined by (6) and the path of integration is the ray $\arg a(T-Z) = 0$.

The method of successive approximations shows that (7) has a solution with simple asymptotic properties. Let us consider a strip Δ_0 in Δ, namely $A \leqq X$, $B_1 \leqq \Im(aZ) \leqq B_2$ and suppose that $|W_0(Z)| < K$ in Δ_0. In the notation of §2 we obtain for the first approximating function

$$|W_1(Z)-W_0(Z)| \leqq \frac{1}{r}\int_0^\infty |\sin rt\, F(Z+e^{-i\phi}t)\, W_0(Z+e^{-i\phi}t)|\, dt$$
$$< \frac{K}{r}\int_0^\infty M(u+t)\, dt.$$

Denoting the last integral by $N(u)$ we obtain by complete induction

$$|W_n(Z)-W_{n-1}(Z)| < \frac{K}{n!}\left[\frac{N(u)}{r}\right]^n.$$

Hence $W_n(Z)$ converges uniformly in the strip Δ_0 towards an analytic function $W(Z)$ such that

$$|W(Z)-W_0(Z)| < K\left[e^{\frac{1}{r}N(u)} - 1\right] \tag{8}$$

where

$$u = \Re\left[e^{i\phi}Z\right] \text{ and } N(u) = \int_0^\infty M(u+t)dt. \tag{9}$$

Since $e^v - 1 < 2v$ if $0 < v \leqq 1$ we have

$$|W(Z)-W_0(Z)| < 2K\frac{N(u)}{r} \tag{10}$$

if $u \geqq u^*$ and $N(u^*) = r$.

Similar relations hold in Δ outside of the strip Δ_0. Thus, if Z lies above Δ_0 but still in Δ, we have

$$|e^{aiZ}\,[W(Z)-W_0(Z)]| < K_+\left[e^{\frac{1}{r}N(u)} - 1\right] \tag{11}$$

where K_+ is an upper bound of $|e^{aiZ} W_0(Z)|$ in the region under consideration. We can obtain similar bounds for the variation of $W'(Z)$.

4. The case $a = 0$ is handled in the same way. The associated integral equation becomes

$$W(Z) = W_0(Z) + \int_Z^\infty (T-Z)\, F(T)\, W(T)\, dT, \tag{12}$$

where $W_0(Z) = C_1Z + C_2$. For the sake of simplicity we shall assume the path of integration is arg $(T - Z) = 0$.

We observe that a constant K can be found such that, if $X \geqq A$,

$$|W_0(Z)| < K|Z| = K \sec. \Theta\, X, \quad \Theta = \arg Z.$$

The method of successive approximations shows that

$$|W(Z) - W_0(Z)| < K \sec. \Theta\, [e^{N_2(X)} - 1] \tag{13}$$

where

$$N_2(X) = \int_0^\infty t^2 M(X + t)\, dt. \tag{14}$$

If $X \geqq X^*$, where $N_2(X^*) = 1$, we can write $2\,N_2(X)$ instead of the bracket. Further, if $W_0(Z)$ reduces to a constant we can leave out the factor sec. Θ and replace $N_2(X)$ by

$$N_1(X) = \int_0^\infty t\, M(X + t)\, dt. \tag{15}$$

5. The question of uniqueness can be handled in two steps. First, it is shown that any solution of (7) is bounded on a ray arg $(T - Z_0) = -\phi$ provided the integral on the left hand side of (7) converges for $Z = Z_0$ and, in addition, the solution in question is finite on any finite portion of the ray. Secondly, it is shown that two bounded solutions must be identically equal. The case $a = 0$ requires a slightly modified argument since $|W(Z)| = O(|Z|)$.

6. It is easily shown that the solution $W(Z)$ of the integral equation (7) satisfies the differential equation (3) no matter how $W_0(Z)$ is chosen subject to the condition of satisfying the asymptotic differential equation (6). Conversely, every solution of the differential equation (3) will satisfy an integral equation of type (7) for some particular choice of $W_0(Z)$ among the solutions of (6), provided we restrict Z to remain in Δ. Thus there is a (1,1) correspondence between the solutions of (3) and those of (6). This correspondence is such that $W(Z)$ is asymptotic to $W_0(Z)$ in the sense defined by formulas (8–11). If $a = 0$ we have a similar situation.

In particular, it follows that the oscillatory properties of $W(Z)$ and $W_0(Z)$ are asymptotically equal in Δ. Thus if $W_0(Z)$ has no zeros or extrema of

large modulus in Δ the same holds for $W(Z)$. This requires either $a = 0$ or $a \neq 0$ with $c_1 c_2 = 0$. The formulas of §§ 3 and 4 enable us to determine regions in which $W(Z)$ cannot vanish. If, for example, $a \neq 0$, $c_1 = 0$, $c_2 = 1$, formula (11) shows that $W(Z) \neq 0$ in that part of Δ where $u \geqq u_0$ assuming $N(u_0) = r \log 2$.

On the other hand, if $a \neq 0$ and $c_1 c_2 \neq 0$, then $W(Z)$ will oscillate in Δ. Again formulas (8) and (11) give information about the zeros of $W(Z)$. The results which we read off from these formulas, using the theorem of Rouché, can be expressed as follows:

To $W_0(Z) = \sin a(Z - Z_0)$ and a preassigned positive ϵ correspond a positive u_ϵ and an integer n_ϵ such that $W(Z)$ has one and only one zero in each of the circles $\left| Z - Z_0 - n\frac{\pi}{a} \right| < \epsilon$, $n > n_\epsilon$, which lie in the half-plane $u = \mathfrak{R}[e^{i\phi} Z] \geqq u_\epsilon$, and no other zero in that part of Δ which belongs to the half-plane mentioned.

7. We can of course translate this discussion from the Z-plane to the z-plane or to the surface $\mathfrak{R}$ using the inverse of transformation (2). Everything goes over smoothly, analytic representation, estimation of bounds and distribution of the zeros.

It remains to point out some cases in which the present discussion is applicable. Suppose, for example, that z_s is an isolated singular point such that $K(z)$, $G(z) = (z - z_s)^{\alpha_\nu} P_\nu(z - z_s)$, where $\nu = 1, 2$, $\alpha_2 - \alpha_1$ is an integer $n \leqq -2$ and $P_1(0) = 1$, $P_2(0) = g \neq 0$.

I. If $\alpha_2 - \alpha_1 < -2$ we have an irregular singular point of finite rank. We find $a = 1$ and $M(u) = M(X) = m X^{-2}$, $m > 0$. If $\alpha_1 + \alpha_2 = 0$ or $3\alpha_1 - \alpha_2 - 4 = 0$, $M(X)$ will decrease still more rapidly.[1]

II. If $\alpha_2 - \alpha_1 = -2$ we have a regular singular point. In order that the method here proposed shall be applicable it is necessary and sufficient that g shall lie outside of a region in the g-plane bounded by the negative real axis, the line $\mathfrak{R}(g - c) \leqq 0$, $\mathfrak{I}(g - c) = 0$ and the line-segment from 0 to c, where $c = \frac{1}{4}(\alpha_1 - 1)^2$. In that event, $a^2 = 1 - \frac{c}{g}$ and $M(u) = m \exp\left[-\mathfrak{R}\left(\frac{e^{i\phi}}{\sqrt{g}} \right) u \right]$ where $-\frac{\pi}{2} < \arg \sqrt{g} < +\frac{\pi}{2}$.[2]

The condition on g of course excludes singular points which, from the Fuchsian point of view, are very simple, no modification of the method seems to be able to take care of all these excluded cases. On the other hand, the scope of the method can be enlarged by conformal representation. If, for example, there exists a transformation $z^* = f(z)$ carrying z_s into a point z_s^* which falls under one of the enumerated categories, then the method under discussion applies directly to z_s just as easily as to z_s^*.

It may be possible to cover $\mathfrak{R}$ completely by regions D satisfying the conditions of §2. In that event the method here proposed gives a complete

solution of the integration problem as well as of the distribution problem for the zeros from the local point of view.

[1] The writer has studied the somewhat more general case $|F(Z)| < m|Z|^{-1-\gamma}$, $\gamma > 0$, in several papers; see, e.g., *Trans. Amer. Math. Soc.*, **23**, 1922 (350–385); *Ibid.*, **26**, 1924, (241–248). Cf. also Hoheisel, *J. Math., Berlin*, **153**, 1924 (228–248).

[2] See a forthcoming paper by the writer in the *Ark. Matem., Stockholm*, which, however, treats only a few special cases.

TOPOLOGICAL INVARIANTS OF MANIFOLDS

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In a 3 dimensional manifold M, two sensed, 2-cycle S_i, and S_j taken in the order written, intersect in a 1-cycle which we shall denote by $S_i\, S_j$. With suitable conventions as to the sense of the cycle of intersection, $S_i\, S_j = -S_j\, S_i$. Three sensed 2-cyles S_i, S_j, S_k, taken in the order written, (likewise a 1-cycle and a 2-cycle), intersect in a certain number of points. We denote the difference between the number of positive points of intersection and the number of negative points by the integer a_{ijk} and write

$$S_i\, S_j\, S_k = S_i(S_j\, S_k) = (S_i\, S_j)S_k \sim a_{ijk}$$

Now let

$$(C^i) \qquad C^1, \ldots, C^P, C^{P+1}, \ldots, C^Q$$

be a basic system of 1-cycles such that every other 1-cycle of the manifold is homologous to a linear combination of the cycles C^i. If the system (C^i) is in reduced form, the first P cycles of the set are linearly independent; the cycle C^{P+i} satisfies a single independent homology

$$(1) \qquad T_i\, C^{P+i} \sim 0 \text{ (not summed for } i)$$

where T_i is the ith coefficient of torsion of the manifold. The Betti numbers of the manifold are $(P_1 - 1) = (P_2 - 1) = P$. If a general 1-cycle is expressed in terms of the cycles C^i by the homology

$$C \sim a_i\, C^i \text{ (summed for } i)$$

the coefficient of C^{P+i} is only determined to a multiple of the coefficient of torsion T_i, that is to say, it may always be reduced modulo T_i.

Dual to the linearly independent cycles C^i,$(i = 1, \ldots, P)$, are certain 2-cycles

$$(S_i) \qquad Si \ldots, S_P$$

such that

$$C^i S_j \sim \delta^i_j \quad (\delta^i_i = 1;\ \delta^i_j = 0, i \neq j). \tag{2}$$

The cycles S_i form a basis for the 2-cycles of the manifold M.

Let us express the intersection of a pair of 2-cycles $S_i S_j$ as a linear combination of the fundamental 1-cycles.

$$S_i S_j \sim a_{ijk} C^k \tag{3}$$

By combining (2) and (3), we have

$$S_i S_j S_k \sim a_{ije} C^e S_k \sim a_{ije} \delta^e_k \sim a_{ijk},$$

which shows that the tensor a_{ijk} associated with the intersection of a pair of 2-cycles is the same as the one associated with the intersection of three, a fact which I failed to observe in a previous note to these PROCEEDINGS (**10**, No. 3, p. 99).

Let us denote by J the sub-field of the field of all linear 1-cycles determined by the cycles of intersection of pairs of 2-cycles; that is to say, the field composed of all 1-cycles of the form

$$a^{ij}_{st} S_i S_j,$$

where the coefficients a^{ij}_{st} are integers. We shall say that a 1-cycle C is homologous to zero modulo J

$$C \sim 0 \pmod{J},$$

if it lies in the field J. We may then determine a reduced basis for the 1-cycles, modulo J, by reducing to normal form the system (C^i) conditioned by the homologies

$$\begin{aligned} T_i C^{p+i} &\sim 0 \\ a_{ijk} C^k &\sim 0 \pmod{J} \end{aligned}$$

Let the reduced system be

$$C^1, \ldots, C^\pi, C^{\pi+1}, \ldots, C^\kappa$$

with

$$\phi_i C^{\pi+i} \sim 0 \pmod{J} \quad \text{(not summed for } i\text{)}$$

We then have what may be called Betti numbers π and coefficients of torsion ϕ_i modulo J. These new numbers are independent of the regular Betti numbers and coefficients of torsion of Poincaré, as may be seen by comparing two examples cited in a previous note to these PROCEEDINGS (**10**, No. 3, 101).

The extension of the above to n dimensions is immediate.

THE NEUTRAL BATH AND ITS RELATION TO BODY HEAT

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Basal metabolism measurements have passed from the stage of physiological curiosities to pathological necessities, and hence the definition of basal metabolism, the prerequisite conditions and the techniques involved in its measurement, and above all the interpretation of the findings of basal metabolism measurements challenge attention today as never before. Complete muscular repose in the lying position, the post-absorptive state, i.e., at least 12 hours after the last meal (which should not be too rich in protein), and normal body temperature have regularly been recognized as essentials. The effect of drowsiness or sleep has been specially recognized by Dr. T. M. Carpenter, and all workers have held the environmental temperature for the subject at such a point that it would not be so warm as to produce discomfort or so cold as to induce shivering.

Recently Lefèvre in Paris[1] has asserted that modern basal metabolism technique is fundamentally wrong in that the pure basal metabolism cannot be measured unless the subject is immersed in a neutral water bath of 35° to 36°C, to prevent loss of heat to the environment. This challenge of the generally accepted technique, accompanied by the fact that no other therapeutic procedure has been so effective in the handling of cases of mental disturbance, led to an investigation of certain physiological effects of the prolonged neutral bath. Four well-trained subjects, three men and one woman, were used. The basal metabolism, with due observance of all the standard conditions, was first measured with the subject lying, normally clothed, covered with one thickness of light-weight cotton blanket. The room temperature during this time was, for the most part, held purposely near 15°C, i.e., lower than the temperature usually obtaining under such measurements in laboratory rooms, hospital wards, or the clinician's office. This was done in order to accentuate the difference in environmental temperature, when subsequent measurements were made in the water bath.

After two or three determinations (each of about 15 minutes' duration) of the basal metabolism in the laboratory room at 15°C and after the subject had been lying down for approximately $1^1/_2$ to 2 hours in this environmental temperature, the subject entered the bath, which varied in temperature from 33° to 38°C. The length of time in the bath was in general not far from 2 hours, during which time several respiration experiments were made. All the subjects found the experiments very agreeable, with a tendency to drowsiness in the bath, in full conformity with the well known sedative effects of the bath in psychiatric procedure.

The latest type of portable respiration apparatus,[2] developed and thoroughly tested in the Nutrition Laboratory, was employed in these measurements, and graphic tracings of the movements of the spirometer bell enabled the determination not only of the oxygen consumption but the respiration rate, the total ventilation of the lungs and the volume of the lung ventilation per respiration. Every detail was so adjusted as to have the only variant the temperature environment of the body. A few records of rectal temperature were also secured, and with the woman subject rather a lengthy series of measurements of the vaginal temperature and the skin temperature were made.

Astonishingly slight changes in the physiological factors measured were noted. The pulse rate, although complicated somewhat by the depressing influence of the inhalation of high percentages of oxygen, showed a tendency in general to increase during the bath. The respiration rate usually increased somewhat in the bath, as did the ventilation of the lungs, although there were wide variations. The volume of ventilation per respiration, however, frequently showed a tendency to be somewhat less in the bath experiments.

Rectal temperatures taken at the end of the bath experiments with two of the men subjects showed no variation from the usual normal rectal temperature. Bath temperatures of from 36° to 38°C resulted in a fairly continuous increase in vaginal temperature throughout the entire period of immersion. Only in baths at 35°C or below was seeming constancy noted in the trunk temperature.

The basis of Lefèvre's criticism is that the body is continually losing heat to the cold (15°C) environment, even though clothed. A knowledge of the skin temperature is, therefore, of unusual importance in this connection. With the thermo-electric technique described in a previous volume of these PROCEEDINGS,[3] a careful study of skin temperatures under the clothing was made with the woman subject before she entered the bath, i.e., in a room at 15°C. Differences in skin temperature ranging from 20.8°C to 34.6°C were found. The pronounced differences in the skin temperature of different parts of the body under the clothing disappeared when the body was immersed in the bath, the skin temperature being higher with higher bath temperatures.

The best index of basal metabolism, in the absence of direct calorimetric measurements, is the oxygen consumption and in all experiments this was most carefully determined. The results, expressed as oxygen consumption per minute reduced to 0°C and 760 mm. pressure, are given in the table herewith, from which it can be seen that there was very good physiological agreement in the basal data with all four subjects, and the values can therefore be accepted as correct indices of the basal metabolism under ordinary conditions. Furthermore, computations of the heat production per 24

hours showed that the measured basal metabolism of these four subjects was normal, as indicated by comparison with well accepted standards.

COMPARISON OF THE OXYGEN CONSUMPTION BEFORE AND DURING IMMERSION IN A NEUTRAL WATER BATH

SUBJECT[1]	DATE 1923–1924	BASAL[2] ROOM TEMP. °C	BASAL[2] OXYGEN CONSUMPTION IN CC. PER MINUTE[3]	BATH TEMP. °C	IN BATH OXYGEN CONSUMPTION IN CC. PER MINUTE[3]
Subject A.					
32 yrs.	Dec. 7	21.0	187 196	37.0	219 210 186
60 kgs.	Dec. 8	15.5	187 190 192	36.5	194 186 191 190
162 cms.	Dec. 10	16.2	199 202 182	36.5	182 200 206 187
	Jan. 3	15.8	193 188 195	35.0	195 195 197
	Jan. 24	14.8	197 199	34.0	194 200 192 196
	Jan. 25	14.9	197 198	33.2	208 201
	Jan. 28	15.1	205 194 191	38.2	193 206
Subject B.					
47 yrs.	Oct. 4	18.2	204 205 197	36.1	208 215 210 202
67 kgs.	Oct. 5	16.2	203 207 200	35.9	211 222 223 224 230
169 cms.	Oct. 6	18.1	196 220 213	35.7	212 213 215 211 236 224
Subject C.					
24 yrs.	Dec. 27	19.3	278 279 289	36.4	313 314 298
91 kgs.	Dec. 28	15.4	276 267 271 268	36.9	301 302 316
178 cms.					
Subject D.					
41 yrs.	April 23	16.4	213 211 219	35.0	258 226 237 238 228
67 kgs.	April 24	17.2	228 215 218 210	35.2	253 237 227 239 242
166 cms.					

[1] Subject A was a woman. Subjects B, C and D were men.

[2] In the basal periods the subject was lying, clothed and covered with a light blanket, and in the post-absorptive condition (i.e., at least 12 hrs. after the last meal).

[3] Computed at 0°C, dry, and 760 mm. pressure.

As can be seen, in all cases the basal values thus obtained were not subsequently lowered by immersion in the bath. Indeed, with the three men there was a definite increase in oxygen consumption in the bath. With the woman the bath temperatures were without significant effect upon the metabolism, although there was certainly no evidence of a decreased metabolism.

Thus the evidence as a whole does not point toward any lowering of metabolism as the result of even a prolonged sojourn in the bath, and it is clear that the heat production under the conditions studied must be entirely independent of the heat lost to the environment, and therefore entirely independent of the surface area.

As a result of these findings it is clear that the influence of the neutral bath (which is of inestimable value in psychiatric clinics) should receive further, most careful physiological study. But whatever changes may be recommended in the present admittedly imperfect routine of making

basal metabolism measurements, certainly the use of the neutral bath is not justifiable.

The complete details of these experiments are soon to be published in the Bulletin de la Société Scientifique d'Hygiène Alimentaire.

[1] Lefèvre, *Bull. Soc. Sci. d'Hygiène Alimen.*, **10**, 595, 606 (1922).
[2] Benedict and Collins, *Boston Med. Surg. J.*, **183**, 449 (1920).
[3] Benedict, Miles and Johnson, *Proc. Nat. Acad. Sci.*, **5**, 218 (1919).

BODY POSTURE AND MINOR MUSCULAR MOVEMENTS AS AFFECTING HEAT PRODUCTION

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Among the many constantly increasing uses of human metabolism measurements is the accumulation of data to compute the total 24-hour heat output of individuals differing in age, weight, height, sex and muscular activity. The 8 to 10 hours of normal bed rest are fairly well represented by the so-called "basal metabolism" measurements. There are many hours, each day, when people are usually sitting quietly in a chair and certain industries call for many hours standing. Very irregular differences in the values for the heat production when the individual is sitting and standing as compared to lying have been reported. Indeed Lefèvre[1] maintains that the heat output is no greater sitting than lying. A study of the metabolism under these three conditions of lying, sitting and standing is justified for several reasons. In the first place, in many studies, particularly on muscular work, some basis for measurement must be employed upon which will be superimposed the particular muscular activity under consideration. Thus, if the activity of walking is to be studied, the metabolism in the standing position would normally be measured first. Secondly, a knowledge of the metabolism in these three positions is important for the computation of the total 24-hour metabolism, for many hours of each day are spent either sitting or standing. Finally, such a study contributes directly to a knowledge of the effect of minor muscular activity upon metabolism, and hence metabolism measurements in these three positions were necessary. Supplementary data giving evidence as to the real necessity of insisting upon complete muscular repose during basal metabolism experiments were secured in studying the effect of a simple arm movement and a simple leg movement, each of which might easily, if not proscribed, enter into any basal metabolism measurement.

In the experiments reported in this paper there were two questions, therefore, to be answered. What is the difference in the heat production when a person is lying quietly, sitting in a comfortable chair, and standing? Secondly, what is the effect of one simple arm or leg movement upon metabolism, and are experiments in which such movements occur vitiated for basal metabolism measurements? To answer these questions the oxygen consumption was measured under standard basal conditions, varied only by changes in position and in the superimposition of a simple, definite arm or leg movement.

Two subjects were used, one a professional artist's model (a woman), the other one of the writers. The usual portable respiration apparatus[2] of the Nutrition Laboratory was employed, which measures exclusively the oxygen consumption but which records the respiration rate graphically. All the comparisons between lying and sitting and lying and standing were made upon the same day, the subjects being clothed as ordinarily. The room temperature was not far from 20°C. After several basal metabolism periods in the lying position, the subject then sat or stood, as the case might be, and usually two or more periods were immediately made.

A comparison of the oxygen consumption per minute in the lying and sitting positions is given in Table 1, in which it can be seen that with the change from the lying to the sitting position there was a definite increase

TABLE 1

COMPARISON OF THE OXYGEN CONSUMPTION PER MINUTE IN THE LYING AND SITTING POSITIONS

SUBJECT AND DATE	LYING (AWAKE) NUMBER OF PERIODS	LYING (AWAKE) AVERAGE O_2 C.C.	SITTING NUMBER OF PERIODS	SITTING AVERAGE O_2 C.C.	PER CENT DEVIATION DUE TO CHANGE IN POSITION
Subject A.					
Dec. 1, 1920	4	180	2	191	+ 6.1
Dec. 3, 1920	4	173	2	186	+ 7.5
Dec. 6, 1920	2	180	2	188	+ 4.4
Dec. 17, 1920	5	188	2	196	+ 4.3
Jan. 19, 1921	7	195	2	216	+10.8
Jan. 26, 1921	3	188	2	195	+ 3.7
Jan. 27, 1921	2	187	3	192	+ 2.7
Nov. 3, 1921	3	194	3	193	− 0.5
Nov. 4, 1921	2	200	3	204	+ 2.0
Nov. 5, 1921	2	199	3	203	+ 2.0
Nov. 7, 1921	2	207	2	215	+ 3.9
Nov. 10, 1921	3	202	5	208	+ 3.0
Feb. 8, 1924	2	188	5	194	+ 3.2
Subject B.					
Feb. 29, 1924	3	255	1	264	+ 3.5

in the oxygen consumption with both subjects. Usually this increase was slight, being but about 2 or 3 per cent, although one instance of an increase

of 10.8 per cent was noted. A shorter series of experiments when the subjects were lying and standing gave results indicated in Table 2. Here it is

TABLE 2

COMPARISON OF THE OXYGEN CONSUMPTION PER MINUTE IN THE LYING AND STANDING POSITIONS

SUBJECT AND DATE	LYING (AWAKE) NUMBER OF PERIODS	LYING (AWAKE) AVERAGE O_2 c.c.	STANDING NUMBER OF PERIODS	STANDING AVERAGE O_2 c.c.	PER CENT DEVIATION DUE TO CHANGE IN POSITION
Subject A.					
Dec. 15, 1921	3	191	3	213	+11.5
Dec. 16, 1921	3	198	3	219	+10.6
Dec. 21, 1921	3	191	3	208	+ 8.9
Mar. 29, 1922	4	198	2	213	+ 7.6
Subject B.					
Feb. 29, 1924	3	255	1	317[1]	+24.3

[1] One period immediately following this gave a value of 275 c.c. O_2 with the subject standing (leaning), i.e., an increase of 7.8 per cent.

seen that the increment due to standing was much larger than that due to sitting, ranging from 7.6 to 11.5 per cent with Subject A, while with Subject B in one experimental period the increase was 24.3 per cent. In general it may be considered that the metabolism while standing will be 10 per cent greater than while lying. The magnitude of the increment due to sitting or standing is probably inversely proportional to the skill of the subject in balancing the body and in minimizing extraneous muscular movements.

In a series of experiments where the subject, while lying, raised the hand once every 4 seconds to the forehead, a definite increase in metabolism was noted. A computation of the increase due to each individual movement, however, showed that one such movement per minute was without significance in basal metabolism measurements. On the other hand, when the legs were crossed once every 20 seconds, both subjects showed very considerable increases in metabolism, corresponding to an increase in the oxygen consumption for each leg movement of 11 c.c. Such regular movements would have a measurable influence upon basal metabolism and must be ruled out in basal measurements. These experiments demonstrate clearly that there is no reason to relax the strictness of control of extraneous muscular movements during the measurement of basal metabolism.

The complete details of these experiments are soon to be published in the Bulletin de la Société Scientifique d'Hygiène Alimentaire.

[1] Lefèvre, *Bull. Soc. Sci. d'Hygiène Alimen.*, **10**, p. 595 (1922).

[2] Benedict and Collins, *Boston Med. Surg. J.*, **183**, 449 (1920).

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